

VGP351 – Week 5

⇒ Agenda:

- Quiz #2
- Bounding volumes
 - Axis-aligned bounding boxes
 - Oriented bounding boxes
 - Bounding spheres
- BV hierarchies
- ~~Portal culling~~



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Bounding Volumes

⇒ From Wikipedia:

“...a bounding volume for a set of objects is a closed volume that completely contains the union of the objects in the set.”

⇒ Why is this useful?



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Bounding Volumes

➤ From Wikipedia:

“...a bounding volume for a set of objects is a closed volume that completely contains the union of the objects in the set.”

➤ Why is this useful?

- Can represent complex geometry that would be expensive to test with an approximation that is much cheaper to test
- Visibility, collision detection, etc.



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Desirable BV Characteristics

- ⇒ Inexpensive intersection test
 - BVs are used instead of source geometry to speed up *trivial rejection* (or trivial acceptance) tests



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Desirable BV Characteristics

- ⇒ Inexpensive intersection test
 - BVs are used instead of source geometry to speed up *trivial rejection* (or trivial acceptance) tests
- ⇒ Tight fitting to source geometry
 - If the BV is a poor fit, tests between BVs may result in false positives or false negatives



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Desirable BV Characteristics

- Inexpensive intersection test
 - BVs are used instead of source geometry to speed up *trivial rejection* (or trivial acceptance) tests
- Tight fitting to source geometry
 - If the BV is a poor fit, tests between BVs may result in false positives or false negatives
- Inexpensive to compute
 - If the BV is too expensive to compute, the expense of creating it may cancel the speed-up that it provides



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Desirable BV Characteristics

⇒ Easy to transform

- If the object moves, its BV needs to move. If moving the BV is too expensive, it may cancel out the speed-up.



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Desirable BV Characteristics

⇒ Easy to transform

- If the object moves, its BV needs to move. If moving the BV is too expensive, it may cancel out the speed-up.

⇒ Inexpensive to store

- If the BV requires too much space to store or too much time to access, it can negatively impact performance.



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Axis-Aligned Bounding Box

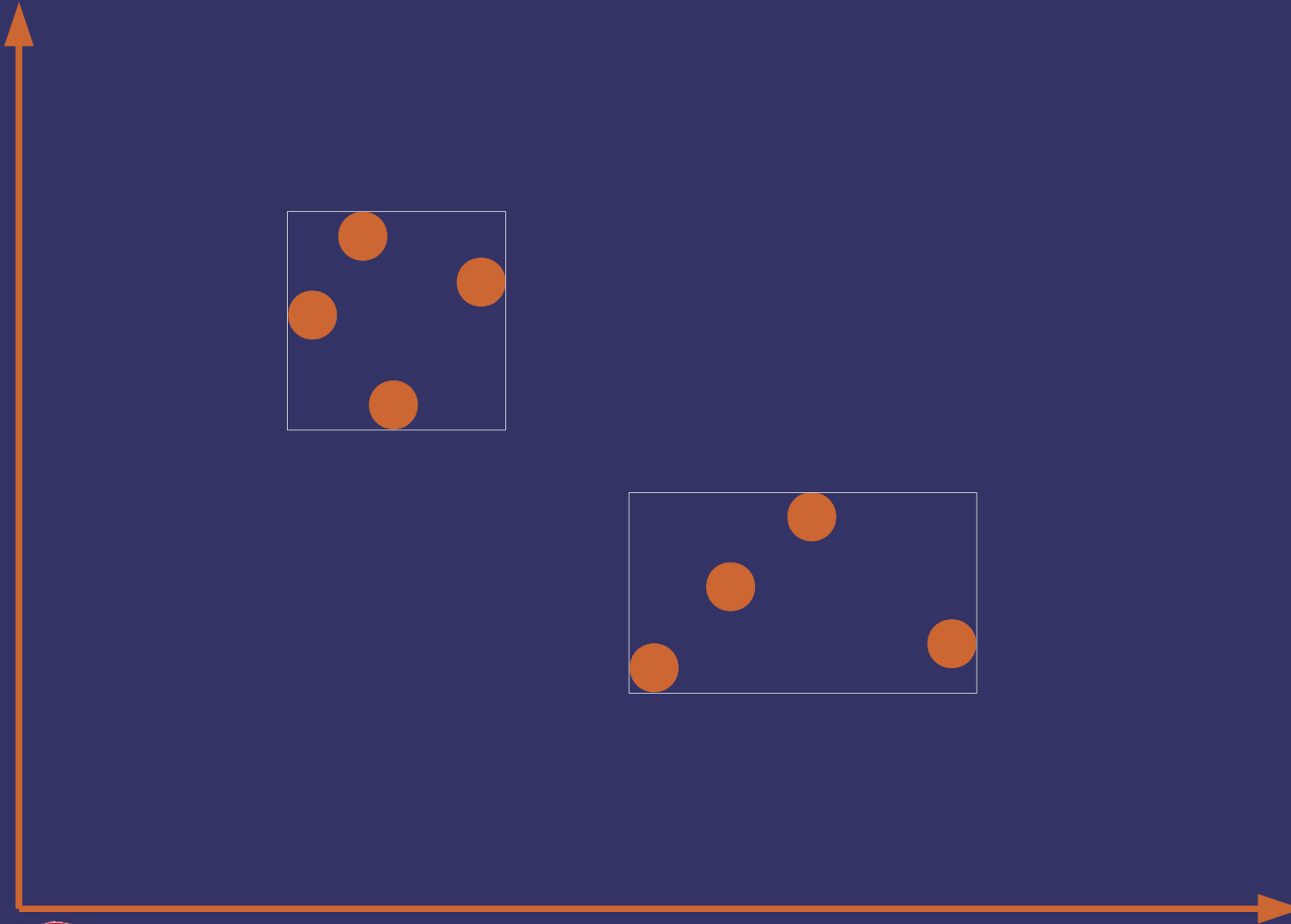
- ⇒ AABB is probably the most common bounding volume
 - Just an n -dimensional box with sides parallel to the principle axes that encloses all the points



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Axis-Aligned Bounding Box



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Axis-Aligned Bounding Box

- ⇒ Three common representations
 - Easy to translate between them
 - Which is used depends on the source data and the usage of the BV



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Axis-Aligned Bounding Box

```
class aabb_min_max {  
    // Points such that for every point P in the  
    // object:  
    //     (min.x <= P.x <= max.x)  
    //     && (min.y <= P.y <= max.y)  
    //     && (min.z <= P.z <= max.z)  
    point min;  
    point max;  
};
```



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Axis-Aligned Bounding Box

```
class aabb_min_diameter {  
    // Points such that for every point P in the  
    // object:  
    //     (min.x <= P.x)  
    //     && (min.y <= P.y)  
    //     && (min.z <= P.z)  
    point min;  
  
    // Dimensions of the box in each direction  
    point diameter;  
};
```



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Axis-Aligned Bounding Box

```
class aabb_center_radius {  
    // Center of the bounding box  
    point center;  
  
    // Radius of the box in each direction  
    point radius;  
};
```



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AABB Creation

⇒ Trivial $O(n)$ problem:

- Scan all points tracking minimum and maximum value in each dimension



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AABB Update

- ⇒ Translation is trivial
 - Rotation is problematic
- ⇒ Three common techniques:
 - Recalculation
 - AABB of an AABB
 - Hill climbing



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AABB Update

- ⇒ Recalculation:
 - Transform source data, calculate new AABB
- ⇒ Advantages / disadvantages?



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AABB Update

⇒ Recalculation:

- Transform source data, calculate new AABB

⇒ Advantages / disadvantages?

- Creates a tight-fitting AABB
- $O(n)$ per transformation is probably much too slow
 - Can speed up by using only points on the convex hull



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AABB Update

⇒ Hill climbing:

- Track the extreme points of the object
- To update, check neighboring points for new extrema

⇒ Advantages / disadvantages?



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AABB Update

⇒ Hill climbing:

- Track the extreme points of the object
- To update, check neighboring points for new extrema

⇒ Advantages / disadvantages?

- Creates a tight-fitting AABB
- Average case performance is good
 - Requires precalculation of convex hull
 - Requires data structure to store connectivity among points on hull



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AABB Update

⇒ AABB of AABB:

- Calculate AABB of base orientation of object
- Apply transformations to object and AABB
- Calculate AABB of transformed AABB

⇒ Advantages / disadvantages?



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AABB Update

⇒ AABB of AABB:

- Calculate AABB of base orientation of object
- Apply transformations to object and AABB
- Calculate AABB of transformed AABB

⇒ Advantages / disadvantages?

- Creates a loose-fitting AABB
- Very fast!

⇒ This is probably the most commonly used technique



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Oriented Bounding Boxes

- ⇒ Arbitrarily oriented box that encloses the object
 - Can lead to much tighter bounding volume
- ⇒ How would you represent an OBB?



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Oriented Bounding Boxes

```
class obb_base_vectors {  
    // Base point of box  
    point base;  
  
    // X, Y, and Z axes  
    point axis[3];  
};
```



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Oriented Bounding Boxes

```
class obb_basis_radius {  
    // Radius in each direction  
    point radius;  
  
    // Transformation to the OBB's coordinate  
    // system  
    matrix basis;  
};
```



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OBB Creation

- ⇒ One common method:
 - Calculate 3D convex hull
 - One of the OBB faces must be coplanar with a face of the convex hull
 - For each face of the 3D convex hull:
 - Project points onto its plane
 - Calculate 2D convex hull
 - Use “rotating calipers” to find minimal bounding rectangle
 - This defines one face of the OBB
 - Calculate distance of farthest point from the convex hull face
 - Use the OBB with the smallest resulting volume



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OBB Creation

⇒ References:

- <http://cbloomrants.blogspot.com/2009/04/04-24-09-convex-hulls-and-obb.html>



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OBB Update

⇒ Trivial!

- Apply transformation to the OBB's basis matrix



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Bounding Spheres

- ⇒ Sphere surrounding the object
 - Ideally it's the *minimal* sphere
 - Representation is trivial
 - Update is trivial



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Bounding Sphere Creation

- Generating a good sphere is a non-trivial exercise
 - Brute-force is $O(n^5)$
 - Statistical methods can produce a good approximation in $O(n)$
 - A recursive method can produce minimum sphere in $O(n)$, but a robust implementation is complex.
 - An iterative approach can get within 5% of minimum in $O(n)$, but has a higher constant factor.



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Bounding Sphere Creation

- Generating a good sphere is a non-trivial exercise
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 - A recursive method can produce minimum sphere in $O(n)$, but a robust implementation is complex.
 - An iterative approach can get within 5% of minimum in $O(n)$, but has a higher constant factor.

We won't talk about these methods today



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Bounding Sphere Creation

⇒ Brute-force:

- A plane is defined by 3 non-collinear points
- A sphere is defined by 3 points on a plane and one point not on the plane
 - i.e., a tetrahedron
- Consider the sphere defined by all combinations of 4 non-coplanar points, keep the smallest that contains all the points.



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Bounding Sphere Creation

⇒ Ritter's algorithm:

- Given an initial guess that is too small, can find bounding sphere within 10% of minimum
- Easy to understand and easy to implement
 - I did a version in 68000 assembly language many years ago



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Bounding Sphere Creation

```
void bounding_sphere(Sphere &sphere, vector *p, unsigned num)
{
    float r_squared = sphere.radius * sphere.radius;

    for (unsigned i = 0; i < num; i++) {
        const vector d = p[i] - sphere.center;
        const float dist_squared = d.dot3(d);

        if (dist_squared > r_squared) {
            const float dist = sqrt(dist_squared);
            const float r = (sphere.radius + dist) / 2.0f;
            const float k = (r - sphere.radius) / dist;

            sphere.radius = r;
            sphere.center += d * k;
            r_squared = r * r;
        }
    }
}
```



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Bounding Sphere Creation

⇒ What's the big assumption?



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Bounding Sphere Creation

⇒ What's the big assumption?

- That we have a good way to come up with an initial sphere
 - The initial sphere must be a little bit too small
 - The better the initial sphere, the better the final sphere



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Bounding Sphere Creation

⇒ What's the big assumption?

- That we have a good way to come up with an initial sphere
 - The initial sphere must be a little bit too small
 - The better the initial sphere, the better the final sphere

⇒ Apply the algorithm iteratively

- Generate a sphere from an AABB
- Apply Ritter's algorithm
- Shrink the output sphere
- Apply again adding the points in random order

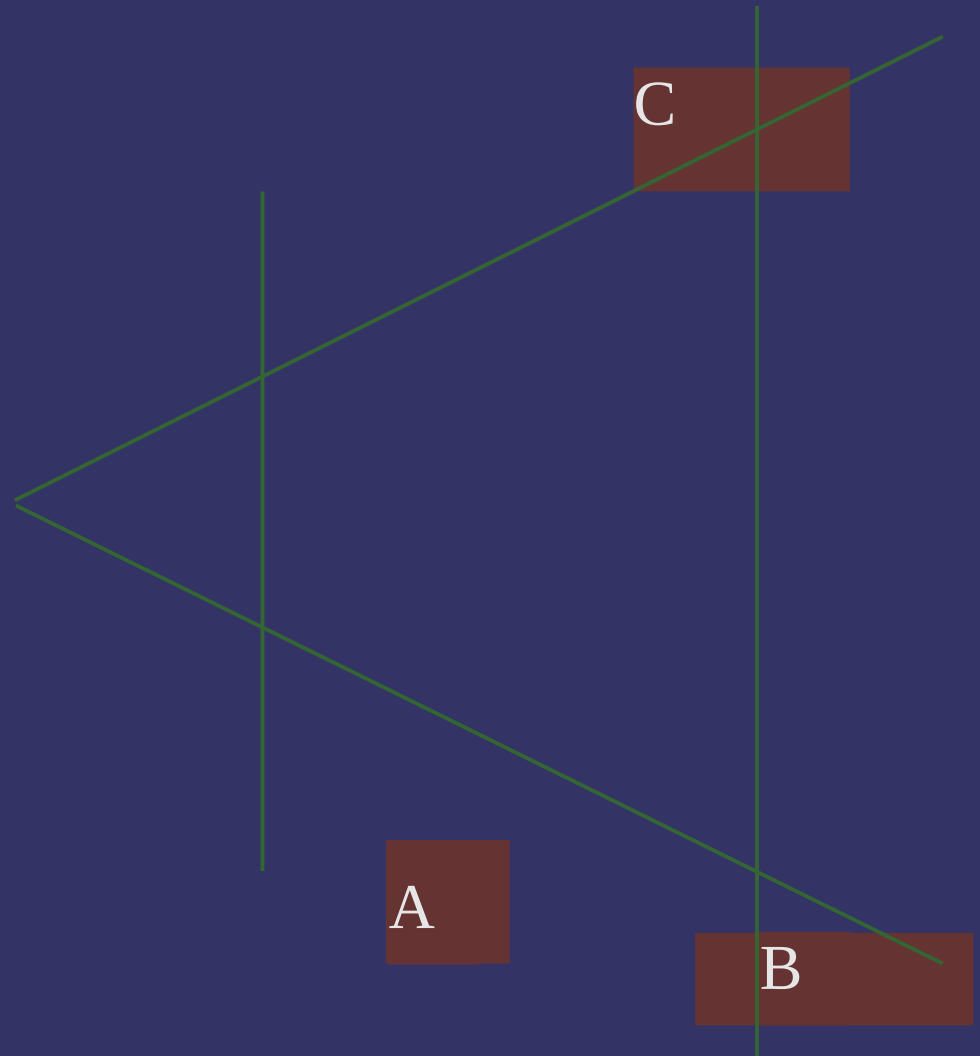


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AABB / Frustum Intersection

- ⇒ Test each corner of the box. If all corners are outside the frustum, then box is outside.

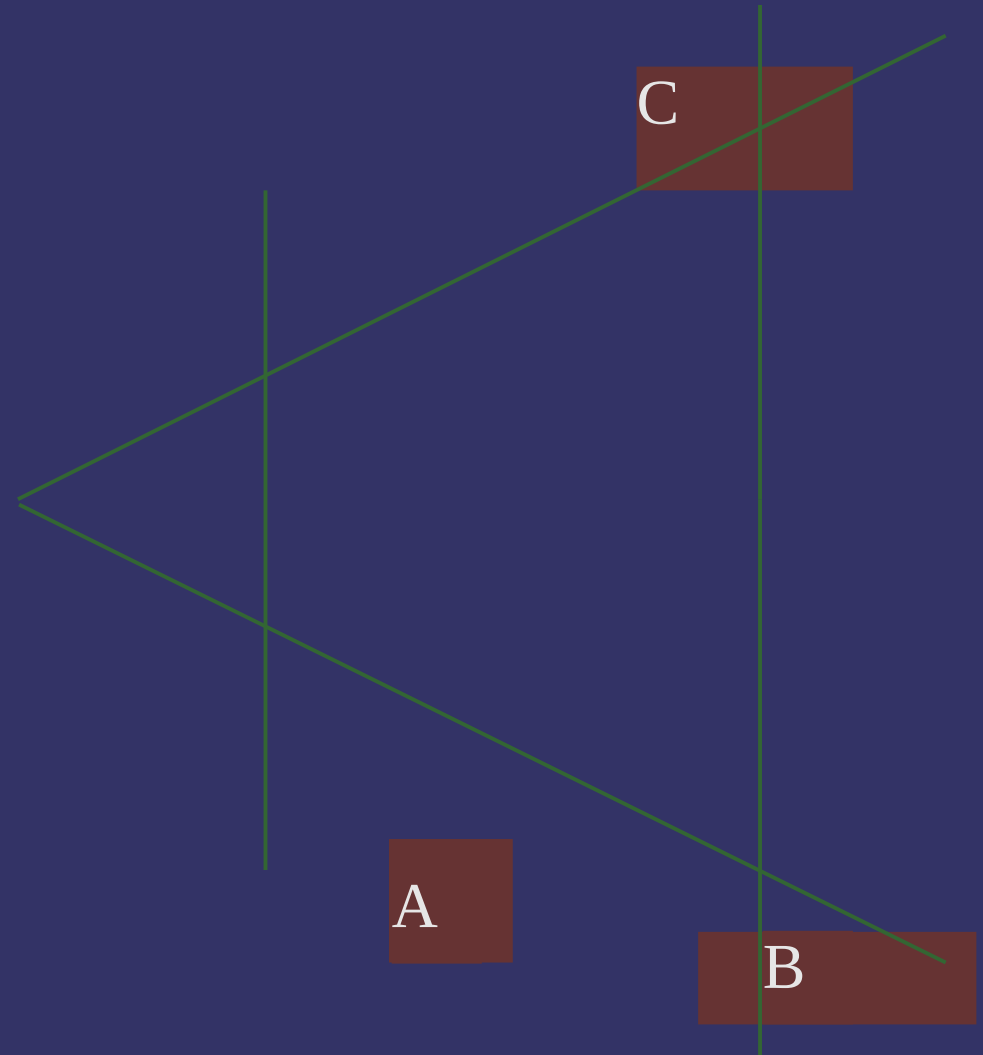


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AABB / Frustum Intersection

- ~~Test each corner of the box. If all corners are outside the frustum, then box is outside. Wrong!!!~~
- If all corners are on positive side of any one plane, then the box is outside.



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AABB / Frustum Intersection

⇒ Can we do better than testing all 8 corners?



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AABB / Frustum Intersection

- ⇒ Can we do better than testing all 8 corners?
 - Pick the “most positive” point and “most negative” point relative to each plane
 - Call these the *p-vertex* and the *n-vertex*
 - Test just those points
 - If both are on the same side of the plane, then **all** of the points must be on that same side



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AABB / Frustum Intersection

- ⇒ Finding p-vertex and n-vertex:
 - Look at the signs of the components of the plane's normal
 - The signs determine which corner the normal points towards
 - Example: If the normal signs are { +, +, - }, then the p-vertex is { box.radius.x, box.radius.y, -box.radius.z }
 - The n-vertex is always the opposite corner



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AABB / Frustum Intersection

```
int frustum_aabb(Plane *planes, Aabb &aabb)
{
    bool intersect = false;
    for (unsigned i = 0; i < 6; i++) {
        vector vn =
            get_negative_far_point(planes[i], aabb);
        if (vn.dot3(planes[i].n) + planes[i].d > 0)
            return OUTSIDE;

        vector vp =
            get_positive_far_point(planes[i], aabb);
        if (vp.dot3(planes[i].n) + planes[i].d > 0)
            intersect = true;
    }

    return (intersect) ? INTERSECTING : INSIDE;
}
```



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AABB / Frustum Intersection

⇒ References:

- http://www.ce.chalmers.se/~uffe/vfc_bbox.pdf
- <http://www.ce.chalmers.se/~uffe/vfc.pdf>



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OBB / Frustum Intersection

⇒ Same!

- Transform the frustum to the coordinate space of the OBB
- This effectively transforms the OBB to an AABB



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BV Hierarchies

- Bounding volume containing bounding volumes containing bounding volumes, etc.
 - Arrange the BVs in a tree-like structure
 - Sibling BVs may occupy overlapping space



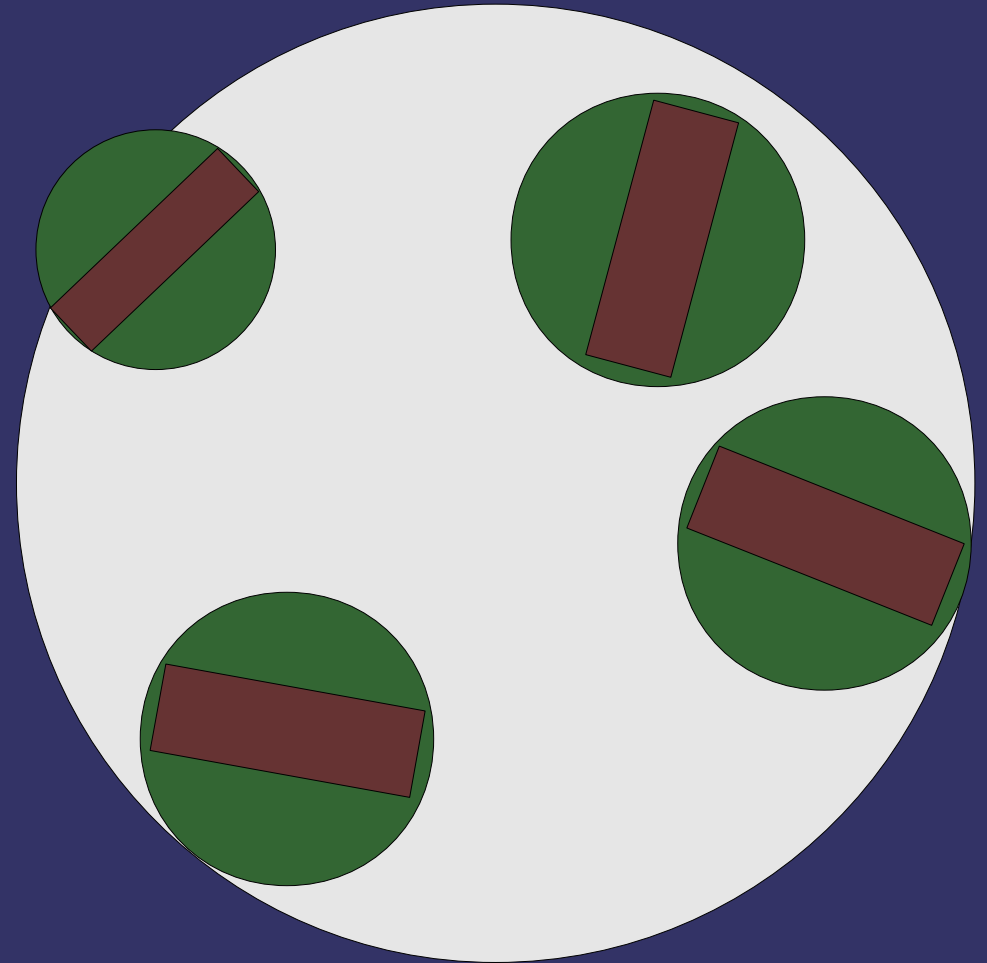
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BV Hierarchies

⇒ Parent-child property:

- Each parent BV contains its child BVs
- Not required, but makes somethings easier
- Parent BV need only contain objects in child BVs
- Top level circle (right) contains all boxes but not all sub-circles.



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Desirable BVH Characteristics

- ⇒ Nodes within a subtree should be “near” each other
 - Farther down the tree, the nodes should be closer



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Desirable BVH Characteristics

- ⇒ Nodes within a subtree should be “near” each other
 - Farther down the tree, the nodes should be closer
- ⇒ Each node should be tight-fitting
 - Just like non-hierarchical bounding volumes



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Desirable BVH Characteristics

- Nodes within a subtree should be “near” each other
 - Farther down the tree, the nodes should be closer
- Each node should be tight-fitting
 - Just like non-hierarchical bounding volumes
- Nodes near the root are more important than nodes near the leaves
 - Trivial reject (or trivial accept) as many objects as possible as with as little work as possible



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Desirable BVH Characteristics

- ⇒ Minimal overlap of sibling nodes
 - Overlap can force traversal of multiple subtrees



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Desirable BVH Characteristics

- Minimal overlap of sibling nodes
 - Overlap can force traversal of multiple subtrees
- Hierarchy should be balance w.r.t. node structure and content
 - Balanced structure just like regular search trees
 - Balanced content (i.e., number of objects in nodes) allows earlier trivial rejection



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Desirable BVH Characteristics

- Minimal overlap of sibling nodes
 - Overlap can force traversal of multiple subtrees
- Hierarchy should be balance w.r.t. node structure and content
 - Balanced structure just like regular search trees
 - Balanced content (i.e., number of objects in nodes) allows earlier trivial rejection
- Worst-case performance should not be much worse than average-case performance
 - Avoid stuttering framerates



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Desirable BVH Characteristics

- ⇒ Generate *without* human intervention
 - Automatically generate without artist or programmer guiding the process



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Desirable BVH Characteristics

- ⇒ Generate *without* human intervention
 - Automatically generate without artist or programmer guiding the process
- ⇒ Memory overhead should be low
 - Just like non-hierarchical bounding volumes



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BVH Creation

- ⇒ Three common strategies:
 - Insertion
 - Top-down
 - Bottom-up



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BVH Creation – Top-Down

- ⇒ Start with single BV and recursively subdivide
 - Easy to implement
 - Doesn't result in optimal BVH



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BVH Creation – Top-Down

```
BVHNode *build_BVH(Entity *e, int num_e)
{
    BoundingBox *bv = new BoundingBox(e, num_e);
    BVHNode *node = new BVHNode(bv);

    if (num_entity < threshold) {
        node->is_leaf = true;
    } else {
        int first_half_count = divide_entities(e, num_e);
        node->child[0] = build_BVH(& e[0],
            first_half_count);
        node->child[1] = build_BVH(& e[first_half_count],
            num_e - first_half_count);
    }

    return node;
}
```



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BVH Creation – Top-Down

- ⇒ The key element is `divide_entities`
 - As coded, assumes each entity is in exactly one set
 - *Not* the only strategy
- ⇒ How do we decide where to divide the set?



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BVH Creation – Top-Down

- ⇒ Median-cut is a common strategy
 - Select an axis
 - Longest axis of the BV being partitioned is a common choice
 - Project all entities onto this axis
 - Sort projected entities by position
 - First half goes in the first node, second half goes in the second node

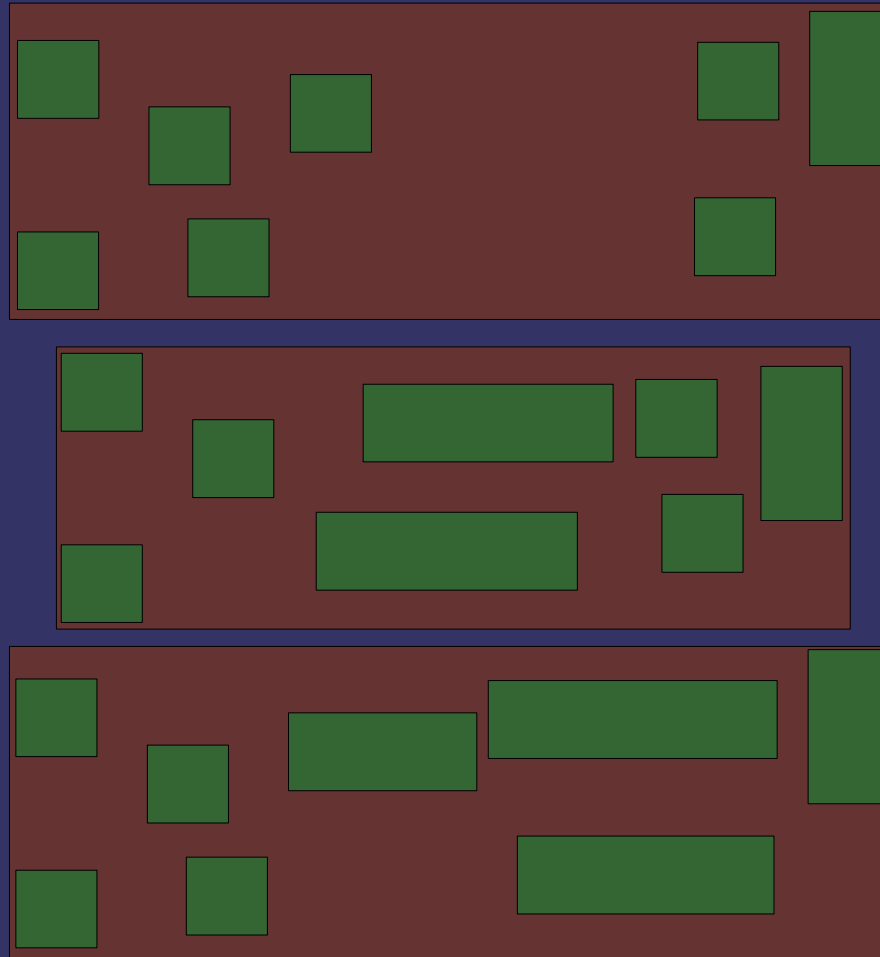


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BVH Creation – Top-Down

- Median-cut is easy to implement, but it poorly partitions some sets:

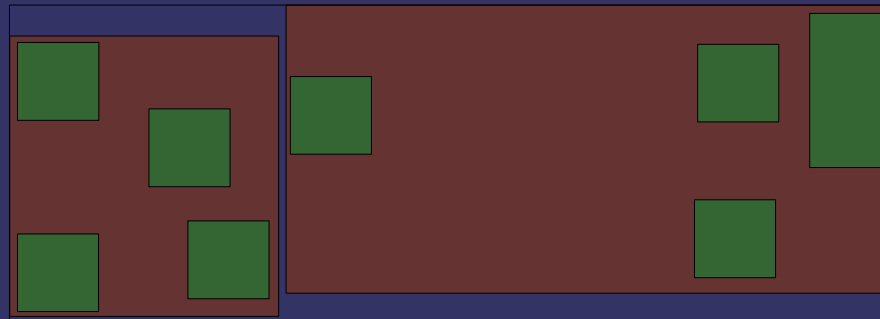


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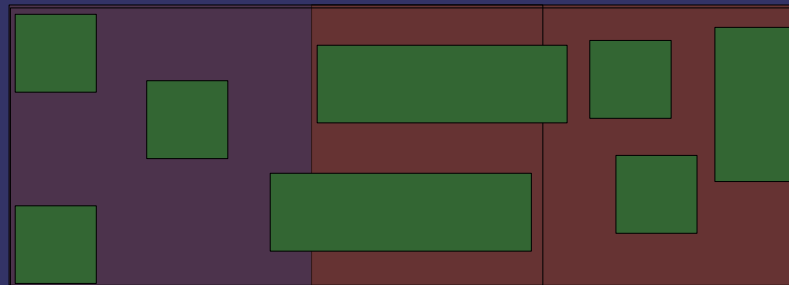
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BVH Creation – Top-Down

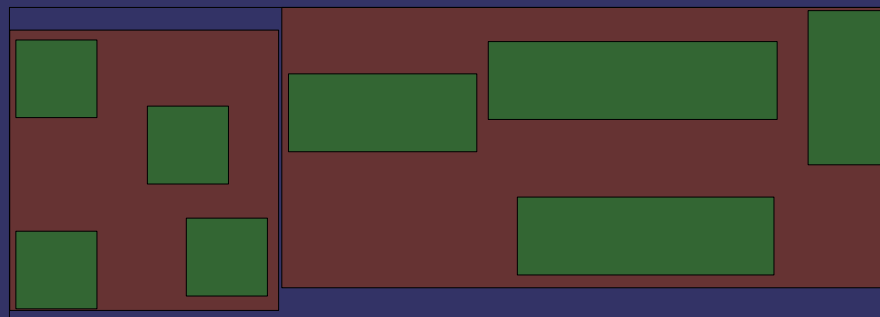
- Median-cut is easy to implement, but it poorly partitions some sets:



Too much empty space



Too much overlap



Unbalanced node sizes



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BVH Creation – Top-Down

⇒ Other heuristics:

- Minimize sum of volumes
- Minimize largest volume
- Minimize overlap volume
- Maximize child node separation

⇒ No single heuristic is perfect

- Implement a primary heuristic and adjust choice if secondary heuristic scores very poorly
- Repeat for all heuristics or until a heuristic passes without adjustment



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BVH Creation – Top-Down

- Infinite number of possible partition axes
 - Similar to the problem of selecting the basis of OBB
- Common choices:
 - Aligned axes of BV
 - Axes of parent BV
 - Axis through most distant points
 - Axis of greatest variance



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BVH Creation – Top-Down

- Once an axis is selected, a split-point must also be selected
 - Median of projected object centroids
 - Mean of projected object centroids
 - Median of projected BV extents
 - Pick best of n evenly spaced points along axis



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BVH Creation – Bottom-Up

- ⇒ Repeatedly merge individual BVs:
 - Create a BV for each object
 - Store in an “active” BV list
 - Select 2 or more BVs to merge
 - Remove old BVs from active list
 - Add new, merged BV to active list
 - Lather, rinse, repeat until only one BV remains
- ⇒ Tradeoffs:
 - Often much, much slower
 - More complex implement



– Usually results in *much* better hierarchies

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BVH Creation – Bottom-Up

- The key element is the algorithm for node selection



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BVH Creation – Bottom-Up

- The key element is the algorithm for node selection
- Obvious, brute-force approach: search active list for pair of nodes that form least-volume BV
 - $O(n^2)$ for the search which is repeated $(n-1)$ times...
 $O(n^3)$ for the lose. :(



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BVH Creation – Bottom-Up

- The key element is the algorithm for node selection
- Obvious, brute-force approach: search active list for pair of nodes that form **least-volume BV**
 - $O(n^2)$ for the search which is repeated $(n-1)$ times...
 $O(n^3)$ for the lose. :(

Other heuristics
can also be used



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BVH Creation – Bottom-Up

- Use the brute-force method as basis for an improved method:
 - For each node, determine the best node for it to pair with
 - Store both nodes with heuristic score in a priority queue
 - Loop, removing the head from the queue:
 - Validate stored size
 - May have changed if either node was already removed
 - If size is still smallest, calculate pairing for new node and add to queue
 - Otherwise, re-insert the original node in the queue



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BVH Creation – Insertion

- Find location to insert node with least cost
 - Heuristic is usually along the lines of volume added to BV and all parent BVs
 - Large objects will be added near the root, small objects will be added near the leaves
 - Far away (isolated) objects will be added near the root



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BVH Creation – Insertion

⇒ Common insertion strategies:

- Depth first:
 - At each step, pick the child with the least cost.
 - Recur on its children
 - Search cost is $O(\ln n)$ with n searches $\rightarrow O(n \ln n)$
- Guided breadth first:
 - Keep track of cost at each visited depth, recur on branch with current best cost
 - Worst-case search cost is $O(n) \rightarrow O(n^2)$
 - Average case is still $O(n \ln n)$
 - Results in much better tree



Uses global information instead of just local information

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Next week...

⇒ Texture mapping, part 1



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